RELATED RATES OF CHANGE

We use the Chain Rule $\{\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}\}$ to relate one rate of change to another rate of change.

Example 1: When a stone is dropped into a still pond of water, a circular ripple is formed. The radius of the circle is increasing at 2 m/s. Calculate the rate at which the area of the circle is increasing when the radius is 8m.

- Find: $\frac{dA}{dt}$
- Use Chain Rule: $\frac{dA}{dt} = \frac{dA}{} \times \frac{}{dt}$
- Since the other variable in the problem is radius(r), the missing term in our Chain rule is dr.
- Therefore $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$
- We have been given $\frac{dr}{dt} = 2\ m/s$. We need to calculate $\frac{dA}{dr}$.
- Since area of a circle is A = πr^2, then $\frac{dA}{dr} = 2\pi r$.
- Rate at which area of circle is increasing = $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ = $2\pi r$ x 2 = $4\pi r$.
- Rate at which area of circle is increasing when the radius is 8 m = $4\pi(8)$= 32π m^2/s.

Example 2: A rubber hot water bottle is being inflated at a steady rate of 1280 cm^3/s. The bottle is spherical, and the formula for the volume of a sphere is V = $\frac{4}{3}\pi r^3$. Calculate the rate at which the radius is increasing at a time of 1 minute after the bottle starts inflating.

- Find: $\frac{dr}{dt}$
- Use Chain Rule: $\frac{dr}{dt} = \frac{dr}{} \times \frac{}{dt}$
- Since the other variable in the problem is volume(V), the missing term in our Chain rule is dV.
- Therefore $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$
- We have been given $\frac{dV}{dt}$ = 1280 cm^3/s. We need to calculate $\frac{dr}{dV}$.
- Since volume of a sphere is V = $\frac{4}{3}\pi r^3$, then $\frac{dV}{dr} = 4\pi r^2$.
- $\frac{dr}{dV}$ is the reciprocal of $\frac{dV}{dr}$, so $\frac{dr}{dV}$ = $\frac{1}{4\pi r^2}$.
- Rate at which radius is increasing = $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$ = $\frac{1}{4\pi r^2}$ x 1280 = $\frac{320}{\pi r^2}$.
- We need a value of r to substitute into $\frac{320}{\pi r^2}$ to give us the rate at which the radius is increasing at time t = 1 minute = 60s.
- Since the volume is increasing at 1280 cm^3/s, at time t = 60, the volume is 1280 x 60 = 76 800 cm^3.
- 76 800 = $\frac{4}{3}\pi r^3$, so solving this gives r = 26.4cm.
- Therefore the rate at which the radius is increasing after 60 s is $\frac{320}{\pi(26.4)^2}$ = 0.146 cm/s.

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